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INTERPOLATION OF VECTOR-VALUED REAL ANALYTIC FUNCTIONS

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Abstract

Let $\omega \subseteq \mathbb{R}^d$ be an open domain. The sequentially complete DF-spaces E are characterized such that for each (some) discrete sequence $(z_n) \subseteq \omega$, a sequence of natural numbers (k_n) and any family $(x_{n,\alpha})_{n\in\mathbb{N},\ |\alpha|\leqslant k_n} \subseteq E$ the infinite system of equations

$$\left(\frac{\partial^{|\alpha|} f}{\partial z^\alpha}\right)(z_n) = x_{n,\alpha} \quad \text{for } n \in \mathbb{N},\ \alpha \in \mathbb{N}^d,\ |\alpha| \leqslant k_n,$$

has an E-valued real analytic solution f.

Introduction

Let us consider an open connected set (*domain*) $\omega \subseteq \mathbb{R}^d$ and an arbitrary discrete sequence $(z_n)_{n\in\mathbb{N}}$ in ω. It is a classical problem to find an analytic function $f : \omega \longrightarrow \mathbb{C}$ which takes prescribed values at (z_n). We are interested in the analogous problem when the given values belong to a fixed locally convex space E.

More precisely, let E be a sequentially complete locally convex space and let $(k_n)_{n\in\mathbb{N}}$ be an arbitrary sequence. We ask when for any family $((x_{n,\alpha})_{\alpha\in\mathbb{N}^d,\,|\alpha|\leqslant k_n})_{n\in\mathbb{N}} \subseteq E$ there is a real analytic E-valued function $f : \omega \longrightarrow E$ with

$$\frac{\partial^{|\alpha|}}{\partial x^\alpha} f(z_n) = x_{n,\alpha} \qquad \text{for every } n \in \mathbb{N},\ \alpha \in \mathbb{N}^d,\ |\alpha| \leqslant k_n. \tag{1}$$

The problem is of special interest if E is a function space. Then we can interpret our problem as if in the infinite system of equations (1) $x_{n,\alpha}$ are scalars depending 'nicely' on some parameter (for instance, holomorphically, smoothly, etc.) and we look for a family of scalar solutions of the system depending as 'nicely' as $x_{n,\alpha}$ on the parameter.

There are two different natural definitions of vector-valued real analytic functions (see [**1**, **21**], cf. [**4**, Definition 7, 8] or [**5**]) and certainly the solution of our problem should depend on the choice of the definition. First, we call a function $f : \omega \longrightarrow E$ *real analytic*, $f \in \mathscr{A}(\omega, E)$, if for every linear continuous functional $y \in E'$, $y \circ f \in A(\omega)$. Secondly, f is called *topologically real analytic*, $f \in \mathscr{A}_t(\omega, E)$, if for every point $x \in \omega$, f develops into a Taylor series convergent around x to f in the topology of E. The relation between these two classes is completely clarified in [**4**] and [**5**], see also Section 1 below. Let us mention only that (contrary to the analogous classes for holomorphic functions) they are different for some Fréchet spaces but for sequentially complete DF-spaces they coincide (see [**4**, Proposition 9]).

It is known, and follows from the theory of π-tensor products, that every Fréchet

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space has an analogous interpolation property for holomorphic functions. Consequently, for Fréchet spaces E each interpolation problem (1) has a solution both in $\mathscr{A}(\omega, E)$ and $\mathscr{A}_t(\omega, E)$. The corresponding problem for complete DF-spaces turns out to be more complicated although in that case $\mathscr{A}(\omega, E) = \mathscr{A}_t(\omega, E)$. The main result of our paper (Theorem 4.1) solves the problem completely, showing that such a space E has the interpolation property if and only if E has the *property* $(\underline{\mathrm{A}})$: whenever (B_n) is a fundamental sequence of Banach discs in E, then there is n such that for every m there are $k, p > 0$ and C with

$$B_m \subseteq rB_n + \frac{C}{r^p} B_k \quad \text{for all } r > 0, \tag{2}$$

which is equivalent to the existence of a fundamental sequence $B_0 \subseteq B_1 \subseteq \ldots$ of bounded Banach discs in E and positive numbers ε_k such that

$$B_k \subseteq rB_0 + \frac{1}{r^{\varepsilon_k}} B_{k+1} \quad \text{for all } k \in \mathbb{N} \text{ and } r > 0.$$

This condition is closely related to the well-known $(\underline{\mathrm{DN}})$ condition (see [**29**, § 29]); in fact, for reflexive E we have $E \in (\underline{\mathrm{A}})$ if and only if $E'_\beta \in (\underline{\mathrm{DN}})$. Therefore, the spaces of germs of holomorphic functions $H(K)$ for natural compact sets $K \subseteq \mathbb{C}^d$ have $(\underline{\mathrm{A}})$ as well as duals of all power series spaces or spaces of Whitney jets over 'nice' sets (see [**29**, 29.12; **32**, **33**]). If we can take $p = 1$ in (2), then E has even (A) and $E'_\beta \in (DN)$.

The crucial step in the proof of sufficiency of Theorem 4.1 is given in Theorem 4.8 showing that $H(\overline{\mathbb{D}}, l_1(I))$ is a 'quotient universal' space for the class of sequentially complete LB-spaces with the property $(\underline{\mathrm{A}})$.

It is known that the analogous interpolation problem for holomorphic functions (that is, analytic of complex variables) has a positive solution for a complete DF-space E if and only if E has (A) [**38**, 4.2, 4.5], see also [**6**]. It is worth pointing out that the difference between (A) and $(\underline{\mathrm{A}})$ (or (DN) and $(\underline{\mathrm{DN}})$) is quite essential. For $H(U)$, U an arbitrary Stein manifold, $H(U)$ has always $(\underline{\mathrm{DN}})$ but it has (DN) if and only if the strong Liouville property holds, that is, every plurisubharmonic function on U bounded from above is constant (cf. [**30**, Proposition 2.1; **36**, p. 262; **40**, 2.3.7]). Analogously, power series spaces $\Lambda_r(\alpha)$ have always $(\underline{\mathrm{DN}})$ but (DN) only if $r = \infty$ [**29**, 29.2, 29.12].

As we explained above, the most important motivation of the paper is provided by the problem of solving equations depending on parameters, see [**4**, **5**, **21**, **22**, **26**, **37**, **38**]. Our research is influenced by the recent extensive research on the space of scalar-valued real analytic functions, which is motivated mostly by its relevance to the theory of partial differential equations, see for example [**2**, **5**, **7**, **9**, **13**, **16**, **18**, **19**, **21**–**25**, **27**, **28**].

The paper is also motivated by the observation that our problem is a question on the lifting of weak*–weak continuous operators $S : E' \longrightarrow \mathbb{C}^{\mathbb{N}}$ with respect to the short exact sequence of the form

$$0 \longrightarrow \ker T \longrightarrow \mathscr{A}(\omega) \xrightarrow{\;T\;} \mathbb{C}^{\mathbb{N}} \longrightarrow 0,$$

where

$$T(f) := \left(\frac{\partial^{|\alpha|} f}{\partial z^\alpha}(z_n) \right)_{n \in \mathbb{N},\ \alpha \in \mathbb{N}^d,\ |\alpha| \leqslant k_n}$$

and, therefore, it is closely connected to the splitting problem for short exact sequences. The study of the splitting problem turned out to be of great relevance for the structure theory of locally convex spaces (see [**29**, § 30, 31; **35**, **36**, **39**]). Even more important is that the splitting results are of great use in many classical questions of analysis, such as when a partial differential or convolution operator has a right linear and continuous inverse, when we can extend functions by means of a linear operator, when we can solve (systems of) equations *nicely* depending on parameters (see, for instance, [**4**, **5**, **10**, **12**, **26**, **32**, **37**]).

The splitting theory seems to be nearly complete for Fréchet spaces (see [**39**]) but for more general spaces appearing in analysis some theory is available only for the space of distributions $\mathscr{D}'$ (see [**11**, **12**]). The present paper is a starting point of such a theory for the space $\mathscr{A}(\omega)$.

1. *Preliminaries and notation*

The space $\mathscr{A}(\omega)$ of real analytic functions $f:\omega \longrightarrow \mathbb{C}$ on an open set $\omega \subseteq \mathbb{R}^d$ is equipped with the topology of the projective limit $\operatorname{proj}_N H(K_N)$, where (K_N) is an increasing sequence of compact sets

$$K_1 \subset\subset K_2 \subset\subset \ldots \subset\subset K_N \subset\subset \ldots \subset \omega, \qquad \bigcup_{N\in\mathbb{N}} K_N = \omega,$$

(a so-called *exhaustion of* ω) and $H(K)$ denotes the space of germs of analytic functions over K with its natural LB-space topology. It is known (see [**28**, Proposition 1.7, 1.2; **14**, 1.2]) that this topology is equal to the inductive limit topology $\operatorname{ind}_U H(U)$, where U runs over all possible open neighbourhoods of ω in $\mathbb{C}^d$ and $H(U)$ denotes the Fréchet space of holomorphic functions on U with the compact open topology. Thus $\mathscr{A}(\omega)$ is a complete, separable, ultrabornological, reflexive, webbed nuclear space with the approximation property (but without a basis [**13**]). The dual of $\mathscr{A}(\omega)$ is the complete LF-space $\mathscr{A}(\omega)'_\beta = \operatorname{ind} H(K_N)'_\beta$. Let E be a sequentially complete locally convex space. The space $\mathscr{A}(\omega, E)$ of E-valued functions, as defined in the introduction, can be represented in various ways:

$$\mathscr{A}(\omega, E) = \mathscr{A}(\omega)\varepsilon E = L(\mathscr{A}(\omega)'_\beta, E) = L(E'_{\mathrm{co}}, A(\omega)),$$

here E'_{co} denotes the space E' endowed with the topology of uniform convergence on compact sets (see [**5**, Theorem 2; **4**, Theorem 16]). For a complete space E we have

$$\mathscr{A}(\omega, E) = \mathscr{A}(\omega)\tilde{\otimes}_\varepsilon E = \mathscr{A}(\omega)\tilde{\otimes}_\pi E.$$

If $T : A(\omega) \longrightarrow \mathbb{C}^{\mathbb{N}}$, $T(f) = (u_n(f))_{n\in\mathbb{N}}$, $(u_n) \subseteq \mathscr{A}(\omega)'$ we define $T\otimes \mathrm{id}_E : \mathscr{A}(\omega, E) \longrightarrow E^{\mathbb{N}}$ depending on the representation of $\mathscr{A}(\omega, E)$ as follows:

(1) for the tensor representation:

$$(T \otimes \mathrm{id}_E)(g \otimes e) = (u_n(g)e)_{n\in\mathbb{N}};$$

(2) for $\mathscr{A}(\omega, E) = L(\mathscr{A}(\omega)'_\beta, E)$:

$$(T \otimes \mathrm{id}_E)(g) = (g(u_n))_{n\in\mathbb{N}};$$

(3) for $\mathscr{A}(\omega, E) = L(E'_{\mathrm{co}}, A(\omega))$:

$$(T \otimes \mathrm{id}_E)(g) = (u_n \circ g)_{n\in\mathbb{N}}.$$

Of course, the above definitions coincide after identifying all these representations and we can use them on $\mathscr{A}_t(\omega, E)$ as well since $\mathscr{A}_t(\omega, E) \subseteq \mathscr{A}(\omega, E)$.

For a sequentially complete space E, it follows from [**4**, Proposition 10(2)(iii)] that $f \in \mathscr{A}_t(\omega, E)$ if and only if for every compact set $K \subseteq \omega$ there is a Banach disc $B \subseteq E$ such that $f \in \mathscr{A}(\omega_K, E_B) = \mathscr{A}_t(\omega_K, E_B)$ for some neighbourhood ω_K of K in ω. If E is a sequentially complete DF-space, then $\mathscr{A}(\omega, E) = \mathscr{A}_t(\omega, E)$ (see [**4**, Proposition 9]). Therefore we get the following proposition.

PROPOSITION 1.1. *A function f belongs to $\mathscr{A}_t(\omega, E)$, E a sequentially complete locally convex space, if and only if there is a sequentially complete DF-space F continuously embedded into E such that $f \in \mathscr{A}(\omega, F) = \mathscr{A}_t(\omega, F)$.*

To clarify the difference between $\mathscr{A}(\omega, E)$ and $\mathscr{A}_t(\omega, E)$ observe that, if $f \in \mathscr{A}_t$, then for any $x \in \omega$ its Taylor series converges on some neighbourhood of a point x for all continuous seminorms p on E, and if $f \in \mathscr{A}$, then for every $x \in \omega$ and every seminorm p there is a neighbourhood of convergence with respect to p.

From now on we assume that ω is a domain and E is sequentially complete.

In a DF-space E, we denote by (B_n) an increasing fundamental sequence of absolutely convex closed bounded subsets and by $(\|\cdot\|_n)_{n\in\mathbb{N}}$ their gauge functionals. For a locally convex space E, we denote by E'_β its strong dual. By *operator*, we always mean a linear continuous one and the space of all operators is denoted by $L(\cdot,\cdot)$.

For smooth functions on $\omega \subseteq \mathbb{R}^d$ we use the standard multi-index notation. Thus if $\alpha = (\alpha_1, \dots, \alpha_d) \in \mathbb{N}^d$, then

$$\frac{\partial^{|\alpha|}}{\partial x^\alpha} f = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \dots \partial x_d^{\alpha_d}} f,$$

where $|\alpha| = \alpha_1 + \dots + \alpha_d$. We denote by $A \subset\subset B$ that A is relatively compact in B.

Other notation is standard. We refer the reader for functional analysis to the books [**29**] and [**17**], and for complex analysis to [**20**].

2. *The properties* ($\underline{\mathrm{A}}$) *and* ($\underline{\mathrm{DN}}$)

We will need some information on the property ($\underline{\mathrm{A}}$) for DF-spaces defined in the introduction and its relation to the better-known property ($\underline{\mathrm{DN}}$) for Fréchet spaces. Let us recall [**29**, § 29] that a Fréchet space F with a fundamental sequence of seminorms $(\|\cdot\|_n)$ has ($\underline{\mathrm{DN}}$) if there is n such that for every m there are $k, \varepsilon \in (0,1)$ and C with

$$\|\cdot\|_m \leqslant C \|\cdot\|_n^\varepsilon \|\cdot\|_k^{1-\varepsilon}.$$

PROPOSITION 2.1 (cf. [**34**, Lemma 3.1 and Bemerkung 3.1′]). *A sequentially complete DF-space E has ($\underline{\mathrm{A}}$) if and only if there is a bounded set $B \subseteq E$ such that for every $x \in E$ there are $k, p > 0$ and $r_0 > 0$ such that*

$$\sup_{r \geqslant r_0} r^p d_k(x, rB) < +\infty, \tag{3}$$

where d_k denotes the distance given by the gauge functional of B_k.

Proof. Necessity. Since $x \in E$ there is m such that $x \in B_m$ and, by ($\underline{\mathrm{A}}$), we get

$$x \in rB_n + \frac{C}{r^p} B_k \quad \text{for } r > 0$$

which implies the condition for $B = B_n$.

Sufficiency. If $r^p d_k(x, rB) < C$ for $r \geqslant r_0$ then $x \in rB + (C/r^p)B_k$ for $r \geqslant r_0$. Without loss of generality we may assume that B is a Banach disc, and then $C_n := \bigcap_{s=n}^{\infty}(sB + (n/s^{1/n})B_n)$ are bounded Banach discs. The condition (3) implies that $\bigcup C_n = E$ and, by the Grothendieck factorization theorem, for every m we find k and D such that

$$B_m \subseteq D\left(sB + \frac{k}{s^{1/k}}B_k\right) \quad \text{for } s \geqslant k.$$

Taking $r = Ds$, $B_n \supseteq B$ and $k \geqslant m$ we get the conclusion. □

As obtained for the property (DN) in [**35**, Lemma 1.4] one gets easily by duality, using also [**36**, Lemma 2.4], the following result.

Proposition 2.2. (a) *If a sequentially complete* (DF)*-space E has* ($\underline{\mathrm{A}}$)*, then* $E'_\beta \in (\underline{\mathrm{DN}})$.

(b) *If E is a dual DF-space, that is, a DF-space with a fundamental sequence of convex bounded sets consisting of sets compact in some locally convex topology weaker than the original topology, then* $E'_\beta \in (\underline{\mathrm{DN}})$ *implies* $E \in (\underline{\mathrm{A}})$.

(c) *For a Fréchet space F the following holds:*

$$F \in (\underline{\mathrm{DN}}) \Longleftrightarrow F'_\beta \in (\underline{\mathrm{A}}) \Longleftrightarrow (F'_\beta)'_\beta \in (\underline{\mathrm{DN}}).$$

It is worth pointing out that (c) is also equivalent to $F'_{\mathrm{ind}} \in (\underline{\mathrm{A}})$, for the inductive dual F'_{ind} which need not be equal to F'_β as shown in [**31**, p. 182].

3. *Necessary condition for solving infinite systems of equations*

It will be useful to interpret our interpolation problems in terms of tensor products. To every interpolation problem (1) we may associate an operator $T : \mathscr{A}(\omega) \longrightarrow \mathbb{C}^{\mathbb{N}}$,

$$T(f) := \left(\frac{\partial^{|\alpha|} f}{\partial z^\alpha}(z_n)\right).$$

Then the interpolation problem has a positive solution for $\mathscr{A}(\omega, E)$ if and only if the map $T \otimes \mathrm{id}_E : \mathscr{A}(\omega, E) \to E^{\mathbb{N}}$ is surjective. Analogously, it has a positive solution for $\mathscr{A}_t(\omega, E)$ whenever $(T \otimes \mathrm{id}_E)(\mathscr{A}_t(\omega, E)) = E^{\mathbb{N}}$. The precise meaning of $T \otimes \mathrm{id}_E$ has been explained in Section 1.

Let us observe that for every open neighbourhood Ω of ω in $\mathbb{C}^d$, T is also defined as a map $T : H(\Omega) \longrightarrow \mathbb{C}^{\mathbb{N}}$ and it is surjective whenever Ω is a domain of holomorphy [**20**, 7.2.7]. In the latter case, by the known facts on the projective tensor products, we have

$$T \otimes \mathrm{id}_E : H(\Omega)\tilde{\otimes}_\pi E \longrightarrow E^{\mathbb{N}}$$

is surjective for any Fréchet space E. Since $H(\Omega)\tilde{\otimes}_\pi E \subseteq \mathscr{A}_t(\Omega \cap \mathbb{R}^d, E)$ and since every open set ω in $\mathbb{R}^d$ has a neighbourhood basis in $\mathbb{C}^d$ consisting of domains of holomorphy (see [**8**, Proposition 1; **15**, Corollary II.3.15]) we obtain the following proposition.

Proposition 3.1. *For any Fréchet space E every interpolation problem* (1) *has a solution in* $\mathscr{A}_t(\omega, E)$.

For sequentially complete DF-spaces E the spaces $\mathscr{A}(\omega, E)$ and $\mathscr{A}_t(\omega, E)$ coincide. Nevertheless, the situation is much more complicated than for Fréchet spaces (also in case of holomorphic functions, see [**6**], cf. also [**34**, 3.5; **37**, 2.6; **38**, 4.2, 4.5]). First, we obtain a necessary condition (as we will see later it is optimal).

THEOREM 3.2. *If for a given sequentially complete DF-space E there exists an operator $T : \mathscr{A}(\omega) \longrightarrow \mathbb{C}^{\mathbb{N}}$ such that $T \otimes \mathrm{id}_E : \mathscr{A}(\omega, E) \longrightarrow E^{\mathbb{N}}$ is surjective, then $E \in (\underline{\mathrm{A}})$. In particular, if for at least one sequence $(z_n)_{n\in\mathbb{N}}$ and one sequence $(k_n)_{n\in\mathbb{N}}$ the interpolation problem* (1) *has a solution in $\mathscr{A}(\omega, E)$, then $E \in (\underline{\mathrm{A}})$.*

We need some lemmas.

LEMMA 3.3. *Let E be a sequentially complete DF-space E. Every operator*

$$T : \mathscr{A}(\omega)' \longrightarrow E$$

factorizes through $H(U)'$, where U is some connected complex neighbourhood of ω.

Proof. Since T acts from an LF-space to a union of Banach spaces, for every compact set $K_n \subseteq \omega$, there is a Banach disc $B_n \subseteq E$ such that T maps continuously $H(K_n)'_\beta$ into E_{B_n}. Clearly, we find a complex neighbourhood V_n of K_n such that T maps continuously $H(V_n)'_\beta$ into E_{B_n}. The lemma holds for $U := \bigcup_{n\in\mathbb{N}} V_n$. □

We denote here by $\mathbb{D}$ the unit disc in $\mathbb{C}$.

COROLLARY 3.4. *If $T : \mathscr{A}(\omega) \longrightarrow \mathbb{C}^{\mathbb{N}}$ is an operator with $T \otimes \mathrm{id}_E(\mathscr{A}(\omega, E)) = E^{\mathbb{N}}$ for a sequentially complete DF-space E, then for every sequence $(x_n)_{n\in\mathbb{N}} \subset E$ there is an operator $S : H(\mathbb{D}^d)'_\beta \longrightarrow E$ and a sequence $(y_n)_{n\in\mathbb{N}} \subseteq H(\mathbb{D}^d)'$ such that $S(y_n) = x_n$ for every $n \in \mathbb{N}$.*

Proof. Clearly, for some sequence $(u_n)_{n\in\mathbb{N}} \subseteq \mathscr{A}(\omega)'$, $T(f) = (u_n(f))_{n\in\mathbb{N}}$. By the identification of $\mathscr{A}(\omega, E)$ with $L(\mathscr{A}(\omega)'_\beta, E)$ (see Section 1), the assumption implies the existence of an operator $R : \mathscr{A}(\omega)'_\beta \longrightarrow E$ such that $R(u_n) = x_n$. On the other hand, by Lemma 3.3, R factorizes through $S_1 : H(U)'_\beta \longrightarrow E$. Since $H(U)$ is always isomorphic to a subspace of $H(\mathbb{D}^d)$ (see [**3**], or [**36**, Satz 5.7]), then $H(U)'_\beta$ is a quotient of $H(\mathbb{D}^d)'_\beta$. □

LEMMA 3.5. *Let E be a sequentially complete DF-space and let $\Lambda_1(\alpha)$ be a nuclear power series space of finite type. If for every sequence $(x_n) \subseteq E$ there is a sequence of elements $(y_n) \subseteq \Lambda_1'(\alpha)$ and an operator $S : \Lambda_1'(\alpha) \longrightarrow E$ such that*

$$S(y_n) = x_n \quad \text{for } n \in \mathbb{N},$$

then $E \in (\underline{\mathrm{A}})$.

REMARK 3.6. If we assume that there is a sequence $(y_n) \subseteq \Lambda_1'(\alpha)$ such that for every $(x_n) \subseteq E$ there is an operator S as above, then E has even (A), see [**38**, Proposition 4.5].

Proof of Lemma 3.5. If E does not have $(\underline{\mathrm{A}})$, then, by Proposition 2.1, for every

$n \in \mathbb{N}$ there is x_n such that for every $k, p > 0$, $r_0 > 0$,

$$\sup_{r \geqslant r_0} r^p d_k(x_n, rB_n) = +\infty.$$

We find $(y_n)_{n \in \mathbb{N}}$ and S according to the assumption. It is easy to see that the operator S can be represented as

$$S(y) = \sum_j u_j y^{(j)},$$

where $y = (y^{(j)})_{j \in \mathbb{N}}$ and $(u_j) \subseteq E$ is a fixed sequence such that for every $\rho < 1$ there is $k(\rho)$ such that

$$\sum_j \|u_j\|_{k(\rho)} \rho^{\alpha_j} =: M_\rho < +\infty.$$

Therefore,

$$x_n = S(y_n) = \sum_j u_j y_n^{(j)}.$$

We fix now $n = k(\frac{1}{2})$, $m \in \mathbb{N}$ and ρ, $1/\sqrt{2} < \rho < 1$, such that

$$\|y_n\|^*_{\rho^2} := \sup_j |y_n^{(j)}| \rho^{-2\alpha_j} < +\infty.$$

For $K = k(\rho)$ we get

$$\begin{aligned}
\left\| x_n - \sum_{j=0}^{m-1} u_j y_n^{(j)} \right\|_K &\leqslant \sum_{j=m}^{\infty} \|u_j\|_K |y_n^{(j)}| \\
&\leqslant \|y_n\|^*_{\rho^2} \sum_{j=m}^{\infty} \|u_j\|_K \rho^{2\alpha_j} \\
&\leqslant \|y_n\|^*_{\rho^2} \sum_{j=m}^{\infty} \|u_j\|_K \rho^{\alpha_j + \alpha_m} \\
&\leqslant \|y_n\|^*_{\rho^2} \rho^{\alpha_m} M_\rho = C\rho^{\alpha_m}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\left\| \sum_{j=0}^{m-1} u_j y_n^{(j)} \right\|_n &\leqslant \sum_{j=0}^{m-1} \|u_j\|_n |y_n^{(j)}| \\
&\leqslant \|y_n\|^*_{\rho^2} \sum_{j=0}^{m-1} \rho^{2\alpha_j} \|u_j\|_{k(1/2)} \\
&\leqslant \|y_n\|^*_{\rho^2} \sum_{j=0}^{m-1} (2\rho^2)^{\alpha_j} \|u_j\|_{k(1/2)} (\tfrac{1}{2})^{\alpha_j} \\
&\leqslant \|y_n\|^*_{\rho^2} (2\rho^2)^{\alpha_{m-1}} M_{1/2} = D(2\rho^2)^{\alpha_{m-1}}.
\end{aligned}$$

Let $s \geqslant (2\rho^2)^{\alpha_1}$, we choose m such that

$$(2\rho^2)^{\alpha_{m-1}} \leqslant s \leqslant (2\rho^2)^{\alpha_m}.$$

Let us take $p = -\log\rho/(\log 2 + 2\log\rho)$, then

$$(2\rho^2)^{-p} = \rho$$

and we have

$$(2\rho^2)^{\alpha_{m-1}} \leqslant s, \quad \rho^{\alpha_m} \leqslant s^{-p}.$$

We have proved that

$$x_n \in DsB_n + \frac{C}{s^p}B_K \quad \text{for } s \geqslant (2\rho^2)^{\alpha_1}.$$

Therefore, for $r = Ds$, $r \geqslant D(2\rho^2)^{\alpha_1} =: r_0$ we have

$$\sup_{r \geqslant r_0} r^p d_K(x_n, rB_n) \leqslant CD^p < +\infty,$$

a contradiction. □

Since $H(\mathbb{D}^d) = \Lambda_1(j^{1/d})$, Theorem 3.2 follows from Corollary 3.4 and Lemma 3.5 immediately.

4. *Characterization of vector-valued interpolation*

We will now prove the following theorem.

THEOREM 4.1. *Let E be a sequentially complete DF-space, $\omega \subseteq \mathbb{R}^d$, $(k_n)_{n\in\mathbb{N}} \subseteq \mathbb{N}$ and let $(z_n) \subseteq \omega$ be an arbitrary discrete sequence. Then the following are equivalent:*

(a) *E has property $(\underline{\mathrm{A}})$;*

(b) *for every family $(x_{n,\alpha})_{n\in\mathbb{N}, |\alpha|\leqslant k_n} \subseteq E$ there is a (topological) real analytic function $f : \omega \longrightarrow E$ such that*

$$\left(\frac{\partial^{|\alpha|}}{\partial x^\alpha} f\right)(z_n) = x_{n,\alpha}$$

for $n \in \mathbb{N}, \alpha \in \mathbb{N}^d, |\alpha| \leqslant k_n$.

The implication (b) ⇒ (a) follows from Theorem 3.2. The converse follows from the following sequence of results.

We say that locally convex space E has the (*strong*) *interpolation property* if condition (b) in Theorem 4.1 holds for every open set $\omega \in \mathbb{R}^d$, discrete set $S = \{z_1, z_2, \ldots\} \subset \omega$ and a sequence $(k_n)_{n\in\mathbb{N}} \subseteq \mathbb{N}$ for some $f \in \mathscr{A}(\omega, E)$ ($f \in \mathscr{A}_t(\omega, E)$, respectively). The immediate consequence of the definition is the following lemma.

LEMMA 4.2. *If E has the (strong) interpolation property, F is locally convex and there is a surjective continuous linear map $q : E \longrightarrow F$ then F has the (strong) interpolation property.*

LEMMA 4.3. *Let X be an arbitrary Banach space. The space $\mathscr{A}(\omega_1, X)$ has the interpolation property for every open set $\omega_1 \subseteq \mathbb{R}^s$.*

Proof. By a lemma of Cartan (see [**8**, Proposition 1; **15**, Corollary II.3.15]) every open subset of $\mathbb{R}^d$ has a basis of complex neighbourhoods consisting of domains of holomorphy. Let $\omega \subseteq \mathbb{R}^d$ be an arbitrary open set. We consider the open set $\tilde{\omega} = \omega \times \omega_1 \subset \mathbb{R}^{d+s}$. We find a pseudoconvex open set $\Omega \subset \mathbb{C}^{d+s}$, $\Omega \cap \mathbb{R}^{d+s} = \tilde{\omega}$ so that the functions $x_{n,\alpha} \in \mathscr{A}(\omega_1, X)$ extend to $\Omega \cap (\{z_n\} \times \mathbb{C}^s)$. By use of the Cartan–Oka theory, the restriction map from $H(\Omega)$ to $H(\bigcup_{n\in\mathbb{N}} \Omega \cap (\{z_n\} \times \mathbb{C}^s))$ is surjective. A tensor product argument implies that the same holds for the corresponding spaces of X-valued functions and we find $f \in H(\Omega, X)$ so that $(\partial^{|\alpha|} f/\partial z^\alpha)(z_n, \zeta) = x_{n,\alpha}(\zeta)$ for all $\alpha = (\alpha_1, \ldots, \alpha_d, 0, \ldots, 0)$, $|\alpha| \leqslant k_n$, $n \in \mathbb{N}$, $\zeta \in \mathbb{R}^s$. The function $F \in \mathscr{A}(\omega, \mathscr{A}(\omega_1, X))$, $F(z) := f(z, \cdot)$, $z \in \omega$ solves the problem. □

Lemma 4.4. *For every Banach space X the restriction map*

$$R : \mathscr{A}(\mathbb{R}^2, X) \longrightarrow \mathscr{A}(S^1, X),$$

S^1 the unit circle in $\mathbb{R}^2$, is surjective.

Remark 4.5. The authors are indebted to the referee for simplifying the formula for $P(f)$ below.

Proof of Lemma 4.4. By the properties of tensor products, it suffices to show the result for $X = \mathbb{C}$. For $f \in \mathscr{A}(S^1)$ put

$$F(z) = \sum_{n=-\infty}^{-1} a_n \bar{z}^{|n|} + \sum_{n=0}^{\infty} a_n z^n,$$

where (a_n) are the Fourier coefficients of f. Since $\sup_{n\in\mathbb{Z}} |a_n| R^{|n|} < \infty$ for some $R > 1$, F is the harmonic extension of f to the unit disc and it extends to a real analytic function in a neighbourhood of the unit circle.

We define

$$P(f)(z) := F\left(\frac{2z}{|z|^2+1}\right) = \sum_{n=-\infty}^{1} a_n \bar{z}^{|n|} \left(\frac{2}{|z|^2+1}\right)^{|n|} + \sum_{n=0}^{\infty} a_n z^n \left(\frac{2}{|z|^2+1}\right)^n,$$

where $z \in \mathbb{C}$ is identified with $(x, y) \in \mathbb{R}^2$. Let us observe that the function $r \mapsto 2r/(r^2+1)$, $r > 0$, is positive and has its global maximum equal to 1 at $r = 1$. Therefore $P(f)$ is a well-defined real analytic function on $\mathbb{R}^2$ and $R(P(f)) = f$. □

Remark 4.6. In fact P is a continuous linear right inverse for R, that is, $\mathscr{A}(S^1, X)$ is isomorphic to a complemented subspace of $\mathscr{A}(\mathbb{R}^2, X)$.

Lemma 4.7. *Let X be an arbitrary Banach space. The space $H(\overline{\mathbb{D}}, X)$ of X-valued germs of holomorphic functions over $\overline{\mathbb{D}}$ has the strong interpolation property for every Banach space X.*

Proof. Let us observe that $\mathscr{A}(S^1, X)$, S^1 the unit circle in $\mathbb{C}$, is just the space $\mathscr{A}_{\mathrm{per}}(\mathbb{R}, X) \subseteq \mathscr{A}(\mathbb{R}, X)$ of 2π-periodic real analytic functions. Developing elements in $\mathscr{A}_{\mathrm{per}}(\mathbb{R})$ into Fourier series and developing elements of $H(\overline{\mathbb{D}})$ into Taylor series we may identify these two spaces, so

$$H(\overline{\mathbb{D}}, X) \simeq H(\overline{\mathbb{D}}) \tilde{\otimes}_\pi X \simeq \mathscr{A}_{\mathrm{per}}(\mathbb{R}) \tilde{\otimes}_\pi X \simeq \mathscr{A}_{\mathrm{per}}(\mathbb{R}, X) \simeq \mathscr{A}(S^1, X).$$

Since $\mathscr{A}(\omega, F) = \mathscr{A}_t(\omega, F)$ for every LB-space F, and $H(\overline{\mathbb{D}}, X)$ is an LB-space, our lemma follows from Lemmas 4.2, 4.3 and 4.4. □

Let I be an index set. We consider the space $H(\overline{\mathbb{D}}, \ell_1(I))$ of germs of $\ell_1(I)$-valued functions on $\overline{\mathbb{D}}$. By Taylor series expansion it is isomorphic to the space

$$\Lambda_0^*(I) = \left\{ (x_{n,i})_{n\in\mathbb{N}, i\in I} : \|x\|_t^* = \sum_{n,i} |x_{n,i}| e^{tn} < +\infty \text{ for some } t > 0 \right\}.$$

It is in a natural way an LB-space. Using the Taylor coefficients we may identify $H(\mathbb{D})$ with the projective limit of sequence spaces

$$\Lambda_{\rho_k} := \{x = (x_n)_{n\in\mathbb{N}} : \lim_n |x_n| e^{-\rho_k n} = 0\}$$

equipped with the norm

$$\|x\|_{\rho_k} := \sup_{n\in\mathbb{N}} |x_n| e^{-\rho_k n},$$

where (ρ_k) is a strictly decreasing null sequence of real numbers. After this identification we may also identify $L(H(\mathbb{D}), \ell_1(I))$ with $\Lambda_0^*(I)$, that is, with $H(\overline{\mathbb{D}}, \ell_1(I))$.

THEOREM 4.8. *Let E be a sequentially complete DF-space. Then E has property $(\underline{\mathrm{A}})$ if and only if there is an index set I, so that E is a continuous image of $H(\overline{\mathbb{D}}, \ell_1(I))$.*

Proof. Since $Y := H(\overline{\mathbb{D}}, l_1) \simeq \Lambda_0^*(I)$, it obviously has $(\underline{\mathrm{A}})$. The property $(\underline{\mathrm{A}})$ is clearly inherited by quotients. If E is a continuous linear image of Y then E endowed with the associated ultrabornological topology is a quotient of Y, thus E has $(\underline{\mathrm{A}})$ which proves sufficiency.

Necessity. We find a fundamental sequence $B_0 \subset B_1 \subset \ldots$ of bounded Banach discs in E, so that with $B = B_0$ and suitable $\varepsilon_k > 0$ we have

$$B_k \subset rB + \frac{1}{r^{\varepsilon_k}} B_{k+1} \tag{4}$$

for all $k \in \mathbb{N}, r > 0$.

We put I to be the disjoint union of the sets B_k, $k \in \mathbb{N}$. Then we have for every k a map $q_k \in L(\ell_1(I), E_{B_k})$ which maps the unit ball of $\ell_1(I)$ onto B_k. This gives rise to a surjective continuous linear map q from $\bigoplus_{\mathbb{N}} \ell_1(I)$ onto E.

Let us assume that $\rho_k/(\rho_{k-1} - \rho_k) \leqslant \varepsilon_{k+1}$. Let us take a sequence of points $(z_n) \subseteq \mathbb{D}$ with $e^{-\rho_{k-1}} < |z_k| < e^{-\rho_k}$ and $g \in H(\mathbb{D})$ with $g(z_k) = 0$, $g'(z_k) \neq 0$. Then we obtain an exact sequence

$$0 \longrightarrow H(\mathbb{D}) \xrightarrow{\;j\;} H(\mathbb{D}) \xrightarrow{\;\pi\;} \mathbb{C}^{\mathbb{N}} \longrightarrow 0, \tag{5}$$

where $j(f) = gf$ and $\pi(f) = (f(z_k))_{k\in\mathbb{N}}$. By the nuclearity of $H(\mathbb{D})$, we obtain easily the short exact sequence

$$0 \longrightarrow L(\mathbb{C}^{\mathbb{N}}, \ell_1(I)) \xrightarrow{\;\pi^*\;} L(H(\mathbb{D}), \ell_1(I)) \xrightarrow{\;j^*\;} L(H(\mathbb{D}), \ell_1(I)) \longrightarrow 0,$$

where $\pi^*(T) := T \circ \pi$, $j^*(T) := T \circ j$. After identification of $L(\mathbb{C}^{\mathbb{N}}, \ell_1(I))$ with $\bigoplus_{n\in\mathbb{N}} \ell_1(I)$ we get the exact sequence

$$0 \longrightarrow \bigoplus_{n\in\mathbb{N}} \ell_1(I) \xrightarrow{\;\pi^*\;} \Lambda_0^*(I) \xrightarrow{\;j^*\;} \Lambda_0^*(I) \longrightarrow 0. \tag{6}$$

In order to prove our result it suffices to extend q onto $\Lambda_0^*(I)$, that is, to find an operator $Q : \Lambda_0^*(I) \longrightarrow E$ such that $Q \circ \pi^* = q$.

For that reason we have to find appropriate 'local sequences' of (6). First, we show that, via the sequential representation, (5) is the 'projective limit' of short exact sequences of the form

$$0 \longrightarrow \Lambda_{\rho_k} \xrightarrow{j_k} \Lambda_{\rho_k} \xrightarrow{\pi_k} \mathbb{C}^k \longrightarrow 0, \tag{7}$$

where $j_k(y) := (\sum_{j=0}^{n} y_j g_{n-j})_{n\in\mathbb{N}}$, $\pi_k(x) := (\sum_{n=0}^{\infty} x_n z_j^n)_{j=1}^k$, (g_j) denotes the sequence of Taylor coefficients of g. The only statement which requires a proof is the equality $\ker \pi_k = \operatorname{im} j_k$.

Observe that if $x = (x_n)_{n\in\mathbb{N}} \in \ker \pi_k$, then the series $\sum_{n=0}^{\infty} x_n z^n$ defines a function h holomorphic on an open disc of radius $e^{-\rho_k}$ vanishing at $z_1, \dots, z_k$. We find y as the sequence of the Taylor coefficients of h/g.

Since π_k acts into a finite dimensional space, the sequence (7) splits, and (6) is the projective limit of short exact sequences

$$0 \longrightarrow L(\mathbb{C}^k, \ell_1(I)) \xrightarrow{\pi_k^*} L(\Lambda_{\rho_k}, \ell_1(I)) \xrightarrow{j_k^*} L(\Lambda_{\rho_k}, \ell_1(I)) \longrightarrow 0,$$

where $\pi_k^*(T) = T \circ \pi_k$, $j_k^*(T) = T \circ j_k$. The spaces of operators appearing above can be naturally identified with spaces of matrices and we obtain short exact sequences of the form

$$0 \longrightarrow \bigoplus_{j=1}^{k} \ell_1(I) \xrightarrow{\pi_k^*} \Lambda_{\rho_k}^* \xrightarrow{j_k^*} \Lambda_{\rho_k}^* \longrightarrow 0,$$

where

$$\Lambda_{\rho_k}^* := \left\{ x = (x_{n,i})_{n\in\mathbb{N}, i\in I} : \|x\|_{\rho_k}^* := \sum_{n,i} |x_{n,i}| e^{\rho_k n} < \infty \right\}.$$

As $\Lambda_{\rho_k}^* \cong \ell_1(\mathbb{N} \times I)$ the sequence splits and $\tilde{q}_k := \bigoplus_{i=1}^k q_k \in L(\bigoplus_{j=1}^k \ell_1(I), E_{B_k})$ can be extended to $S_k \in L(\Lambda_{\rho_k}^*, E_{B_k})$, that is $S_k \circ \pi_k^* = \tilde{q}_k$. As $T_k = S_k - S_{k+1}$ vanishes on $\operatorname{im} \pi_k^*$, we find $A_k \in L(\Lambda_{\rho_k}^*, E_{B_{k+1}})$ so that $T_k = A_k \circ j_k^*$.

Now A_k has the form

$$A_k x = \sum_{n,i} a_{n,i}^{(k)} x_{n,i}$$

for $x = (x_{n,i})_{n\in\mathbb{N}, i\in I} \in \Lambda_{\rho_k}^*$, $(a_{n,i}^{(k)}) \subseteq E_{B_{k+1}}$, where

$$\|A_k\| = \sup_{n\in\mathbb{N}, i\in I} \|a_{n,i}^{(k)}\|_{B_{k+1}} e^{-\rho_k n} < +\infty.$$

We apply (4) to

$$a_{n,i}^{(k)} \in \|A_k\| \, e^{\rho_k n} B_{k+1}$$

with

$$r = \frac{2^{-k}}{\|A_k\|} e^{(\rho_{k-1} - \rho_k) n}$$

and obtain

$$u_{n,i}^{(k)} \in 2^{-k} e^{\rho_{k-1} n} B, \quad v_{n,i}^{(k)} \in 2^{k\varepsilon_{k+1}} \|A_k\|^{1+\varepsilon_{k+1}} B_{k+2}$$

so that $a_{n,i}^{(k)} = u_{n,i}^{(k)} + v_{n,i}^{(k)}$. The second inclusion comes from $\rho_k - (\rho_{k-1} - \rho_k)\varepsilon_{k+1} \leqslant 0$. Therefore $U_k x = \sum_{n,i} u_{n,i}^{(k)} x_{n,i}$ defines a map $U_k \in L(\Lambda_{\rho_{k-1}}^*, E_B)$, $\|U_k\| \leqslant 2^{-k}$ and $V_k x = \sum_{n,i} v_{n,i}^{(k)} x_{n,i}$ defines a map $V_k \in L(\Lambda_0^*(I), E)$, with $U_k + V_k = A_k$ on $\Lambda_{\rho_{k-1}}^*$.

We set

$$F_k = A_k - \sum_{j=1}^{k} V_j + \sum_{j=k+1}^{\infty} U_j.$$

Then $F_k \in L(\Lambda^*_{\rho_k}, E)$ and

$$F_k - F_{k+1} = A_k - A_{k+1} + V_{k+1} + U_{k+1} = A_k.$$

Therefore for $Q_k := S_k - F_k \circ j^*_k$ we have $Q_k = Q_{k+1}$ on $\Lambda^*_{\rho_k}$. This means $Qx = Q_k x$ for $x \in \Lambda^*_{\rho_k}$ defines $Q \in L(\Lambda^*_0(I), E)$. Since $Q \circ \pi^* = q$, the map Q is surjective. □

5. *Remarks*

It is easily seen that a product of spaces with the interpolation property has this property as well. Therefore every product of sequentially complete DF-spaces with ($\underline{\mathrm{A}}$) has the interpolation property. On the other hand, the product of spaces having the analogous interpolation property for holomorphic functions has this property as well and, hence, also the strong interpolation property (that is, in $\mathscr{A}_t(\omega, E)$). Summarizing, every product of sequentially complete DF-spaces with (A) has the strong interpolation property.

Corollary 5.1. *Every product E of duals of power series spaces of finite type has the real analytic interpolation property, that is, the interpolation problem* (1) *has a solution in $\mathscr{A}(\omega, E)$ for any domain $\omega \subseteq \mathbb{R}^d$. If the factors of E are duals of power series spaces of infinite type, the solution exists even in $\mathscr{A}_t(\omega, E)$* (*the strong interpolation property*)*, in particular, this holds for $E = \mathscr{D}'$ the space of distributions.*

Instead of the point interpolation problem studied here one could consider the question of extension of vector-valued real analytic functions from real analytic submanifolds which can be expressed in terms of vanishing of the functor Proj^1 for coherent sheaves of real analytic functions on an arbitrary open set $\omega \subseteq \mathbb{R}^d$. Such analogues of Theorem 4.1 (a) ⇒ (b) are also true and will be presented in detail in a forthcoming paper.

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